Calculus:
Comparison of the the Disk/Washer and the Shell Methods

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Prerequisite Material: It is assumed that the reader is familiar with the following:

Method	Axis of Revolution	Formula	Notes about the Representative Rectangle
Disk Method	x-axis	$V = \pi\int_a^b [f(x)]^2 dx$	$f(x)$ is the length dx is the width
	y-axis	$V = \pi\int_c^d [g(y)]^2 dy$	$g(y)$ is the length dy is the width
Washer Method	x-axis	$V = \pi\int_a^b \left([f(x)]^2 - [g(x)]^2\right)dx$	$f(x)$ is the top curve $g(x)$ is the bottom curve dx is the width
	y-axis	$V = \pi\int_c^d \left([f(y)]^2 - [g(y)]^2\right)dy$	$f(y)$ is the right curve $g(y)$ is the left curve dy is the width
Shell Method	x-axis	$V = 2\pi\int_c^d y[g(y)]dy$	y is the distance to the axis of revolution $g(y)$ is the length dy is the width
	y-axis	$V = 2\pi\int_a^b x[f(x)]dx$	x is the distance to the axis of revolution $f(x)$ is the length dx is the width

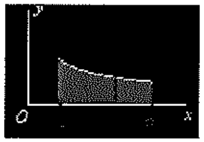
It is important to note that the representative rectangle in the Disk and the Washer Methods are always going to be *perpendicular* to the axis of revolution. With the Shell Method, the representative rectangle will always be *parallel* to the axis of revolution.

Another thing that might help while trying to visualize this type of volume problem, is that the Disk Method is used when the representative rectangle produces a solid that is similar to a plate (no hole in the middle). The Washer Method is used when the rectangle sweeps out a solid that is similar to a CD (hole in the middle). And finally, the Shell Method is used when the rectangle sweeps out a solid that is similar to a toilet paper tube.

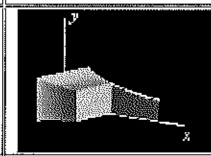
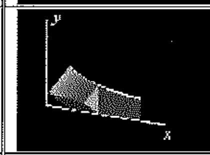
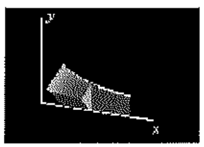
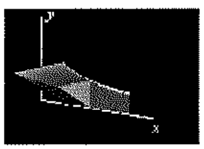
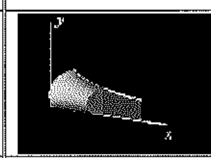
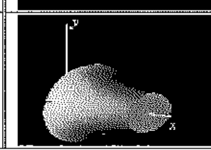
Think about this: Can you see that the Disk Method is just a specific case of the Washer Method?

Created by Sandra Peterson

1. (a)



$y = \frac{1}{\sqrt{x}}$

Solid	Part (b) Representative Cross Section	Part (c) General Formula for Area	Part (d) Expression for Area of Cross Section	Part (e) Integral for Volume	Part (g) Volume
Square		s^2	$\left(\frac{1}{\sqrt{x}}\right)^2$	$\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	0.811 or $\ln\left(\frac{9}{4}\right)$
Equilateral Triangle		$\frac{s^2\sqrt{3}}{4}$	$\frac{\sqrt{3}}{4}\left(\frac{1}{\sqrt{x}}\right)^2$	$\frac{\sqrt{3}}{4}\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	0.351 or $\frac{\sqrt{3}}{4}\ln\left(\frac{9}{4}\right)$
Isosceles Right Triangle #1		$\frac{1}{2}\ell^2$ where ℓ = length of leg of the right triangle	$\frac{1}{2}\left(\frac{1}{\sqrt{2x}}\right)^2$	$\frac{1}{4}\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	0.203 or $\frac{1}{4}\ln\left(\frac{9}{4}\right)$
Isosceles Right Triangle #2		$\frac{1}{2}\ell^2$ where ℓ = length of leg of the right triangle	$\frac{1}{2}\left(\frac{1}{\sqrt{x}}\right)^2$	$\frac{1}{2}\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	0.405 or $\frac{1}{2}\ln\left(\frac{9}{4}\right)$
Semicircle		$\frac{1}{2}\pi r^2$	$\frac{\pi}{2}\left(\frac{1}{2\sqrt{x}}\right)^2$	$\frac{\pi}{8}\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	0.318 or $\frac{\pi}{8}\ln\left(\frac{9}{4}\right)$
Circle (Solid of Revolution)		πr^2	$\pi\left(\frac{1}{\sqrt{x}}\right)^2$	$\pi\int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$	2.548 or $\pi\ln\left(\frac{9}{4}\right)$